Heat and Mass Transfer during welding

- Energy -> Conduction of heat
- Material -> Diffusion of atoms, vacancy, ect…

Diffusion equation

Conduction of heat (1)

- Fourier's Law $\mathbf{q} = -\kappa \, \mathrm{grad} \, T$
  - Heat flux (J/m²s)
  - Thermal conductivity (J/sKm, W/Km)
  - Temperature (K)

- Diffusion equation $\dfrac{\partial T}{\partial t} = -\dfrac{1}{c\rho} \mathrm{div} \, \mathbf{q} = \dfrac{\kappa}{c\rho} \mathrm{div} \left(\mathrm{grad} \, T \right) = D \, \mathrm{div} \left(\mathrm{grad} \, T \right)$
  - Specific heat (J/kgK)
  - Density (kg/m³)
  - Diffusion coefficient (m²/s)

$$D = \frac{\kappa}{c\rho}$$

- Diffusion equation for Cartesian coordinate system

$$\frac{\partial T}{\partial t} = D \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right)$$

Conduction of heat (2)

- Solutions for special boundary conditions
  - Steady state (t=∞), one dimensional

$$\frac{\partial T}{\partial t} = D\left(\frac{\partial^2 T}{\partial x^2}\right) = 0 \quad \Rightarrow \quad T(x) = C_1 x + C_2$$

  - Steady state (t=∞), axially symmetric

$$\frac{\partial T}{\partial t} = D\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) = 0 \quad \Rightarrow \quad T(r) = C_1 \ln r + C_2$$

  - Steady state (t=∞), center symmetric

$$\frac{\partial T}{\partial t} = D\left(\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r}\right) = 0 \quad \Rightarrow \quad T(r) = C_1 \frac{1}{r} + C_2$$

$$\frac{\partial T}{\partial t} = D\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right)$$

Conduction of heat (3)

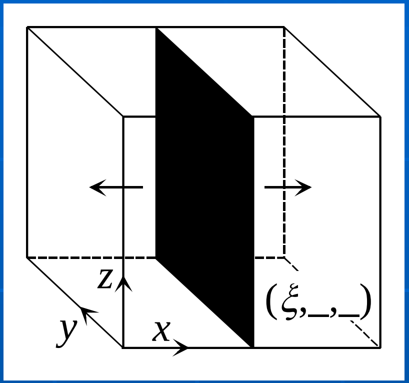
D (m^2/s):thermal diffusion coefficient

ρ (kg/m^3):density, and

c (J/(K kg)):specific.

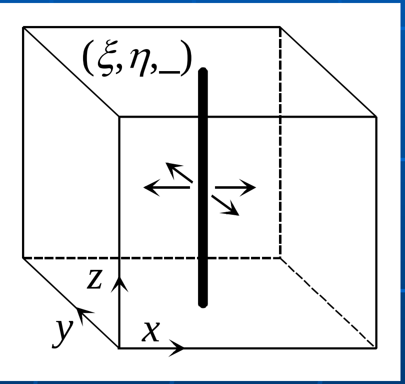
- Basic solutions of diffusion equation
  - one dimensional → area heating; Q^* (J/m^2)

$$T = \frac{Q^*}{c\rho}\frac{1}{\sqrt{4\pi Dt}}\exp\left(-\frac{(x-\xi)^2}{4Dt}\right)$$



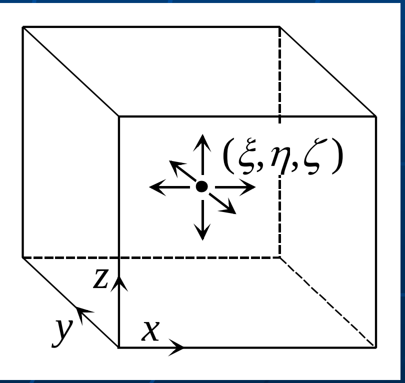
  - two dimensional → linear heating; Q^* (J/m)

$$T = \frac{Q^*}{c\rho}\frac{1}{4\pi Dt}\exp\left(-\frac{(x-\xi)^2+(y-\eta)^2}{4Dt}\right)$$



  - three dimensional → point heating; Q^* (J)

$$T = \frac{Q^*}{c\rho}\frac{1}{(4\pi Dt)^{3/2}}\exp\left(-\frac{(x-\xi)^2+(y-\eta)^2+(z-\zeta)^2}{4Dt}\right)$$



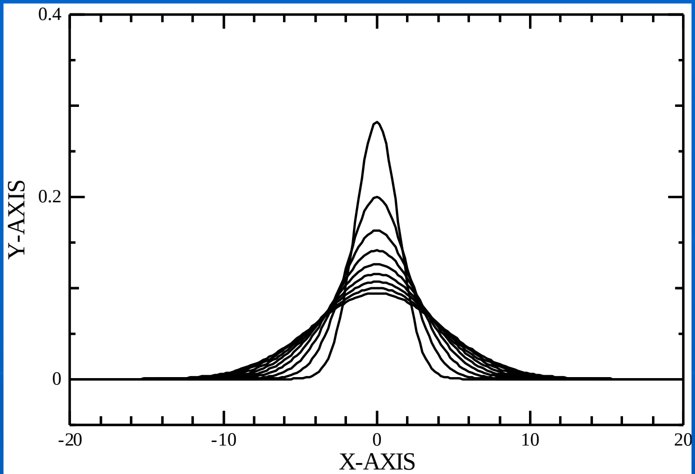
as a function of time *t* and position (*x,y,z*),
for the heat input at the position (ξ, η, ζ).

Conduction of heat (4)

D (m^2/s):thermal diffusion coefficient

ρ (kg/m^3):density, and

c (J/(K kg)):specific.



- Basic solutions of diffusion equation
  - one dimensional → area heating

$$T = \frac{Q^*}{c\rho} \frac{1}{\sqrt{4\pi Dt}} \exp\left(-\frac{(x-\xi)^2}{4Dt} \right)$$

  - two dimensional → linear heating

$$T = \frac{Q^*}{c\rho} \frac{1}{4\pi Dt} \exp\left(-\frac{(x-\xi)^2 + (y-\eta)^2}{4Dt} \right)$$

  - three dimensional → point heating

$$T = \frac{Q^*}{c\rho} \frac{1}{(4\pi Dt)^{3/2}} \exp\left(-\frac{(x-\xi)^2 + (y-\eta)^2 + (z-\zeta)^2}{4Dt} \right)$$

for heat inputs

Q^* (J/m^2)

Q^* (J/m)

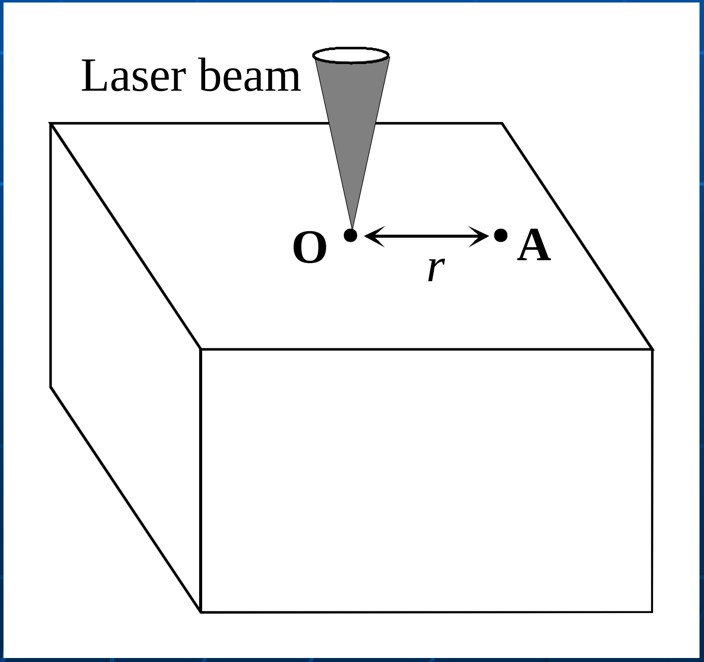
Q^* (J)

as a function of time *t* and position (*x*,*y*,*z*), for the heat input at the position (ξ, η, ζ).

Exercise 1

- Instantly energy is inputted onto a point O on the surface of a semi-infinite body. The temperature is measured at another point A on the surface of the body. The distance OA is r (m) and the energy input is Q (J). Thermal diffusion coefficient, density, and specific heat of the body are D (m^2/s), ρ (kg/m^3), and c (J/(K kg)). Write a function of the temperature change at the point A as a function of time t (s), under the assumption that the material never melt nor evaporate,

  and 100% of the energy input is absorbed in the body.

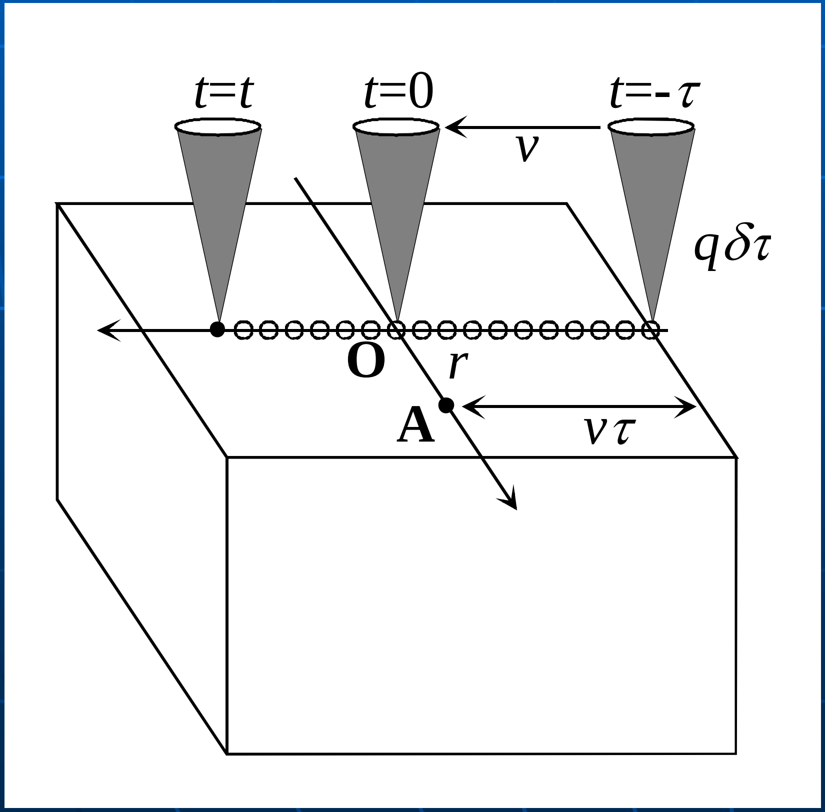


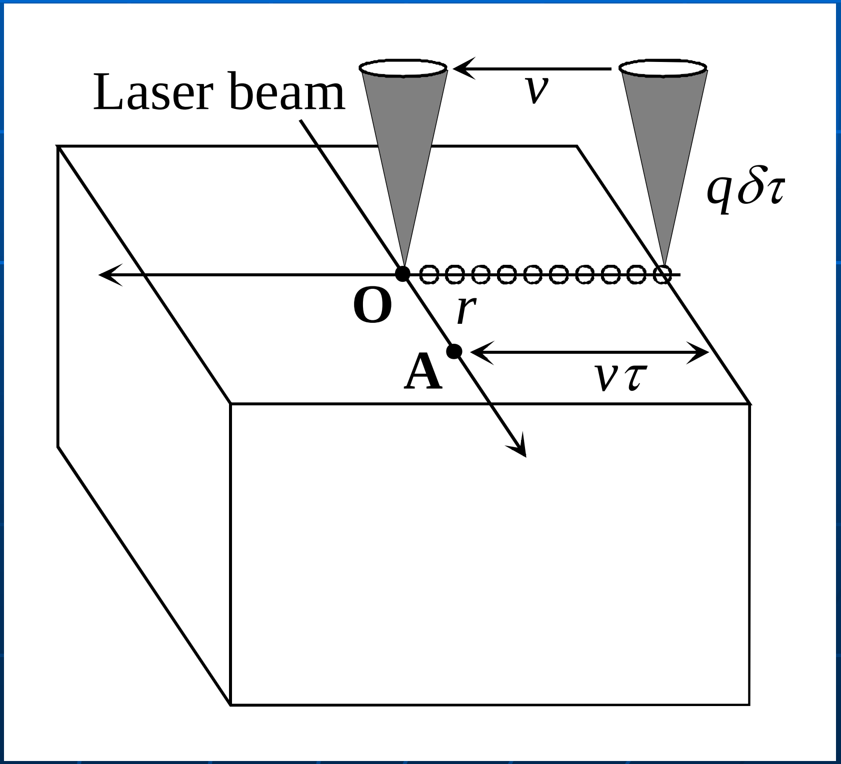
$$\frac{\partial T}{\partial t} = D\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right)$$

$$T = \frac{Q}{c\rho}\frac{1}{(4\pi D t)^{3/2}}\exp\left(-\frac{(x-\xi)^2 + (y-\eta)^2 + (z-\zeta)^2}{4Dt}\right)$$

Exercise 1'

- The heat source moves from $t=t_{start}$ (s) to t_{end} (s). The distance OA is r (m), the energy input is q (J/s), and the velocity is v (m/s). Thermal diffusion coefficient, density, and specific heat of the body are D (m^2/s), ρ (kg/m^3), and c (J/(K kg)). Write a function of the temperature change at the point A as a function of time t (s) in a style of integral.



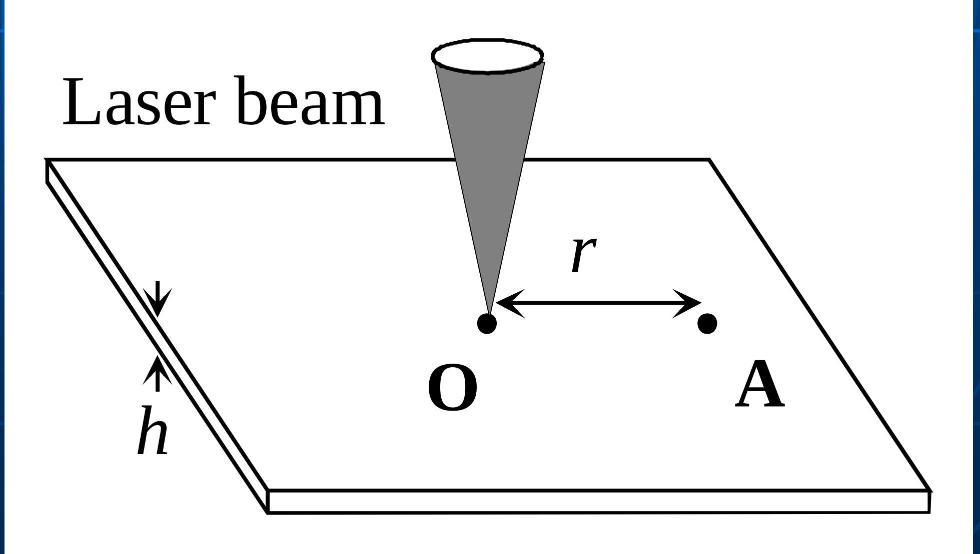


Exercise 2

- Instantly energy is inputted onto a point O on the surface of a thin plate. The temperature is measured at another point A on the plate, whose thickness is h (m). The distance OA is r (m) and the energy input is Q (J). Thermal diffusion coefficient, density, and specific heat of the body are D (m^2/s), ρ (kg/m^3), and c (J/(K kg)). Write a function of the temperature change at the point A as a function of time t (s), under the assumption that the material never melt nor evaporate, and temperature distribution in depth direction can be negligible because of small thickness and no radiant energy loss.
- You can use below eq..

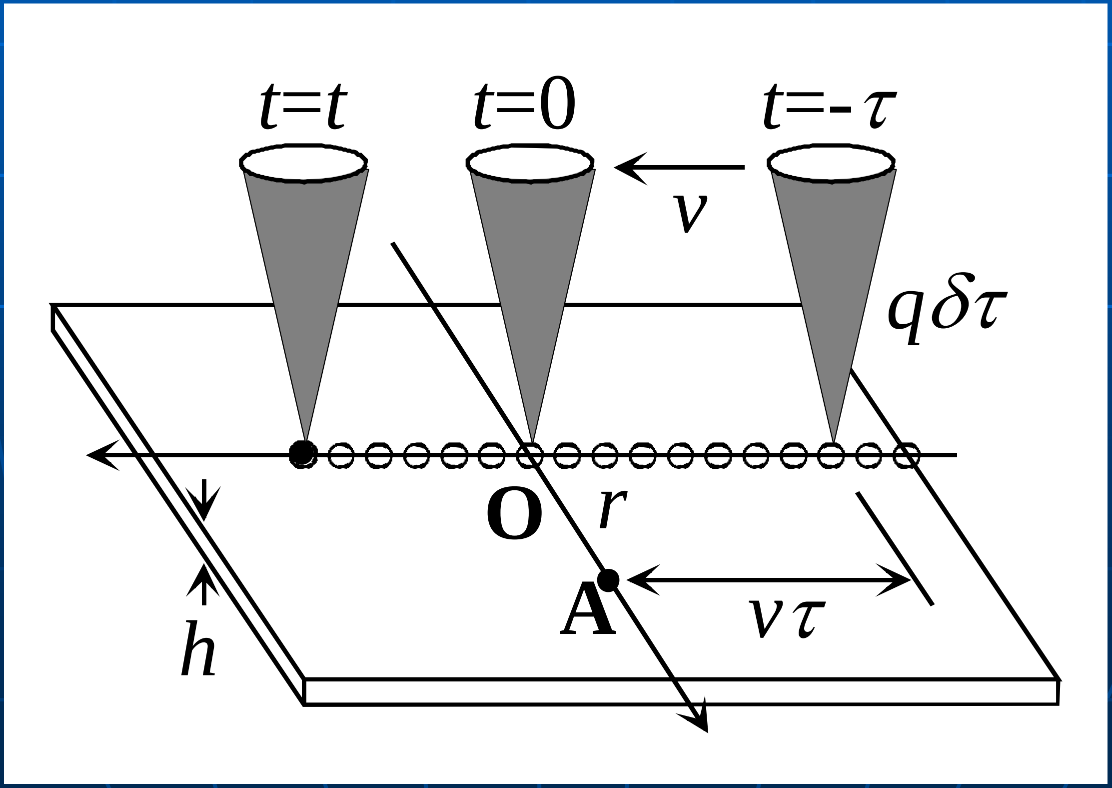
$$T = \frac{Q^*}{c\rho} \frac{1}{4\pi D t} \exp\left(-\frac{(x-\xi)^2 + (y-\eta)^2}{4Dt} \right)$$

where Q^* (J/m) is the heat input per unit length



Exercise 2'

- The heat source moves from $t=t_{start}$ (s) to t_{end} (s). The distance OA is r (m), the energy input is q (J/s), and the velocity is v (m/s). Thermal diffusion coefficient, density, and specific heat of the body are D (m^2/s), ρ (kg/m^3), and c (J/(K kg)). Write a function of the temperature change at the point A as a function of time t (s) in a style of integral.

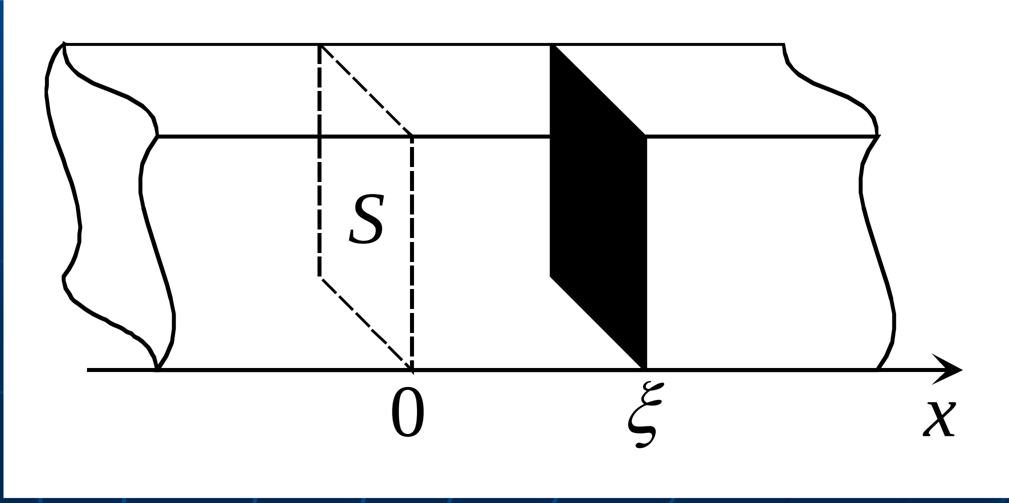


Exercise 3

- Instantly energy Q (J) is inputted at a cross section ($x=\xi$) of the rod. The are of the cross section is S (m^2). Thermal diffusion coefficient, density, and specific heat of the rod are D (m^2/s), ρ (kg/m^3), and c (J/(K kg)). Write a function of the temperature distribution as a function of time t (s) and position x (m), under the assumption that the material never melt nor evaporate, and temperature distribution in the cross section can be negligible because of its small area and no radiant energy loss.
- You can use below eq..

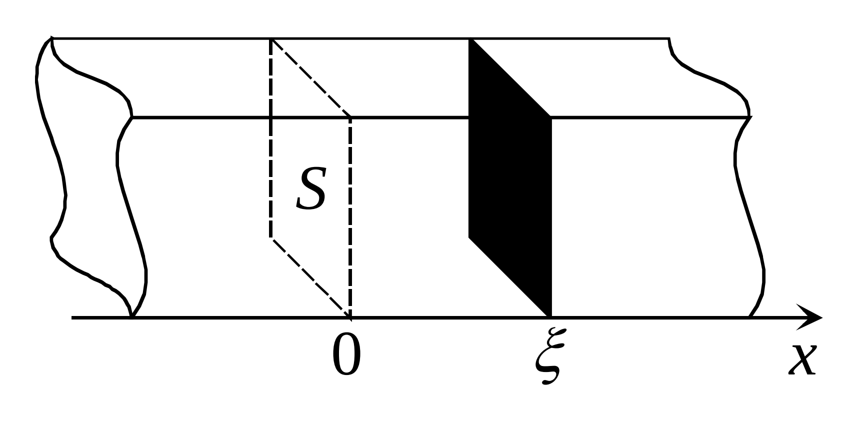
$$T = \frac{Q^*}{c\rho}\frac{1}{\sqrt{4\pi Dt}}\exp\left(-\frac{(x-\xi)^2}{4Dt}\right)$$

where Q^* (J/m^2) is the heat input per unit area



Exercise 3'

- Instantly energy is distributed at the half part (x>0) of the rod. The distributed energy density is q (J/m). Thermal diffusion coefficient, density, and specific heat of the rod are D (m^2/s), ρ (kg/m^3), and c (J/(K kg)). Write a function of the temperature distribution as a function of time t (s) and position x (m).

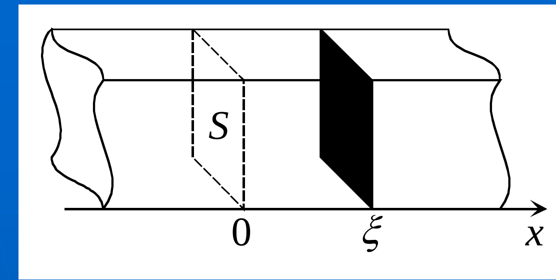


$$T = \frac{(Q/S)}{c\rho}\frac{1}{\sqrt{4\pi Dt}}\exp\left(-\frac{(x-\xi)^2}{4Dt}\right)$$

$$T=\frac{(q/S)}{c\rho}\frac{1}{2}\left\{1+\mathrm{erf}\left(\frac{x}{\sqrt{4Dt}}\right)\right\}=\boxed{\frac{(q/S)}{c\rho}}\frac{1}{2}\,\mathrm{erfc}\left(-\frac{x}{\sqrt{4Dt}}\right)$$

$$\mathrm{erf}(-\alpha)=-\mathrm{erf}(\alpha)$$

$$\mathrm{erfc}(\alpha)=1-\mathrm{erf}(\alpha)=1+\mathrm{erf}(-\alpha)$$



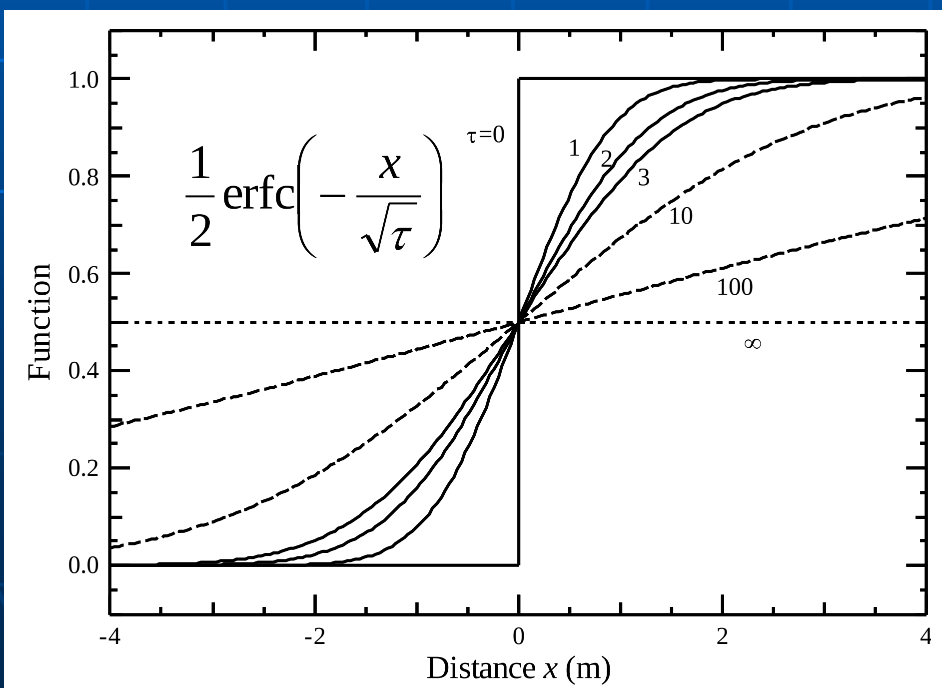
Temperature (K) of the part ($x>0$) at $t=0$!

q/S (J /m^3) :energy input per unit volume

<- q (J/m), and S (m^2)

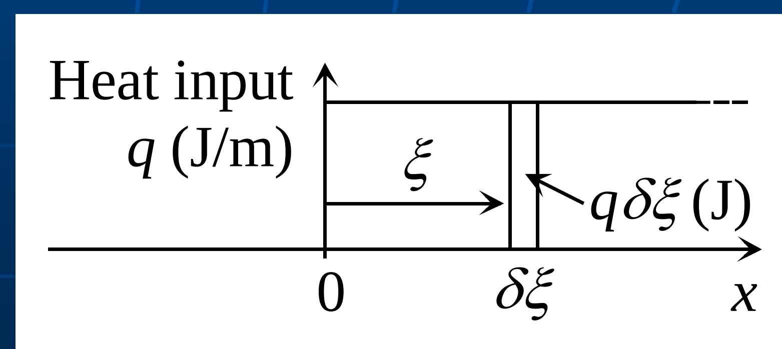
$c\,\rho$ (J/K /m^3) :specific heat per unit volume

<- ρ (kg/m^3), and c (J/(K kg)).



$\frac{x}{\sqrt{4Dt}}$:normalized distance

<- D (m^2/s), and t (s)

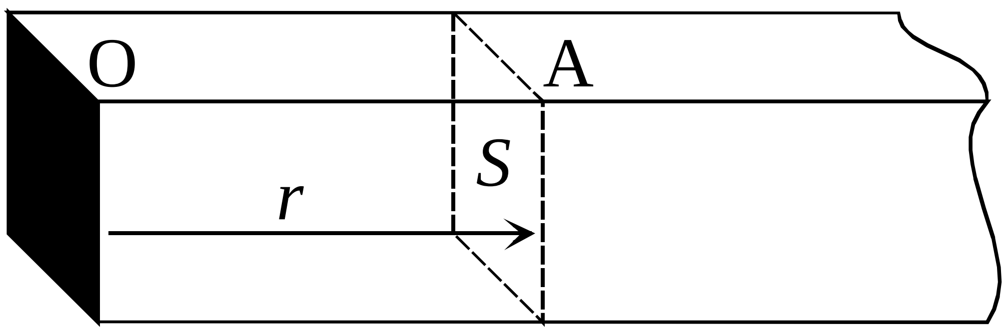


Exercise 4

- Please see below fig.. Instantly energy Q (J) is inputted onto the surface O of a terminal of the rod, whose cross section is S (m^2). The temperature is measured at a plain A. The distance OA is r (m). Thermal diffusion coefficient, density, and specific heat of the rod are D (m^2/s), ρ (kg/m^3), and c (J/(K kg)). Write a function of the temperature change at the plain A as a function of time t (s), under the assumption that the material never melt nor evaporate, and temperature distribution in the cross section can be negligible because of the small cross section and no radiant energy loss.
- You can use below eq..

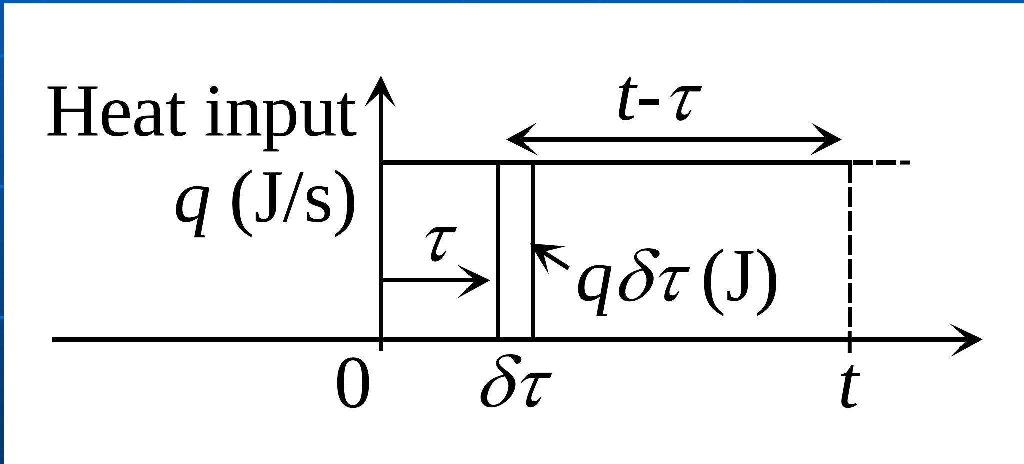
$$T = \frac{Q^*}{c\rho} \frac{1}{\sqrt{4\pi D t}} \exp\left(-\frac{(x-\xi)^2}{4Dt} \right)$$

where Q^* (J/m^2) is the heat input per unit area

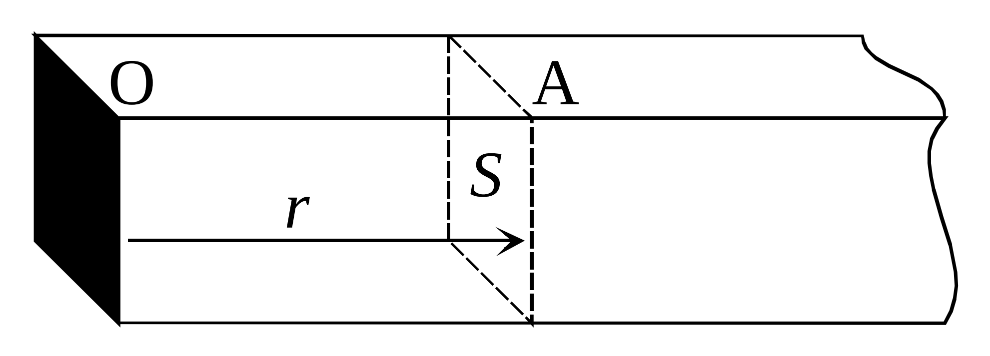


Exercise 4’

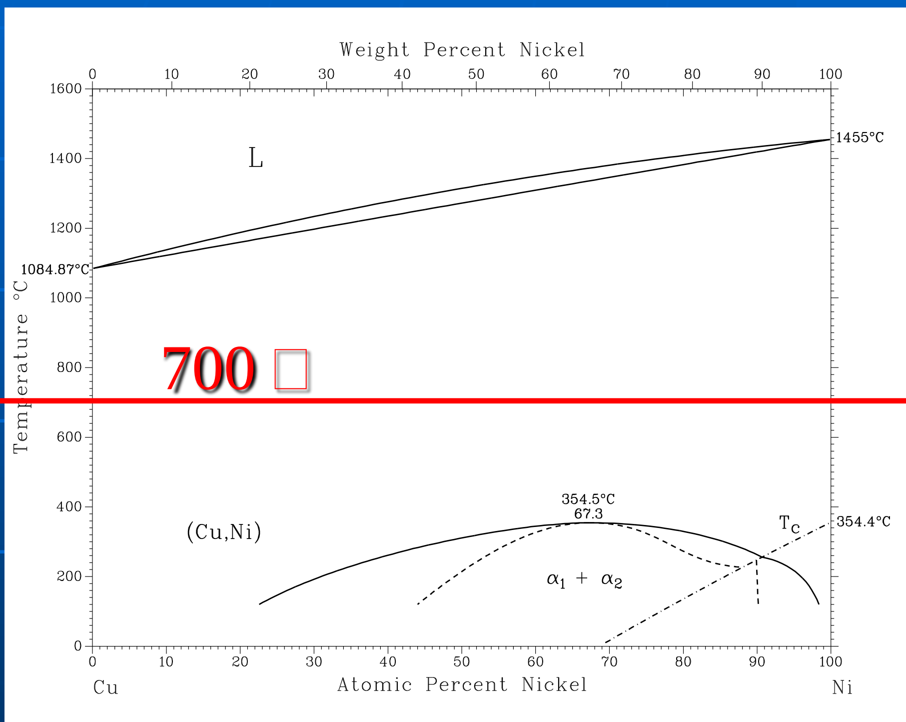
- Continuously, the energy is inputted onto the surface O from t=0 (s). The energy input is q (J/s). The temperature is measured at a plain A. The distance OA is r (m). Thermal diffusion coefficient, density, and specific heat of the rod are D (m^2/s), ρ (kg/m^3), and c (J/(K kg)). Write a function of the temperature change at the plain A as a function of time t (s).



$$T = \frac{2Q/S}{c\rho}\frac{1}{\sqrt{4\pi Dt}}\exp\left(-\frac{r^2}{4Dt}\right)$$

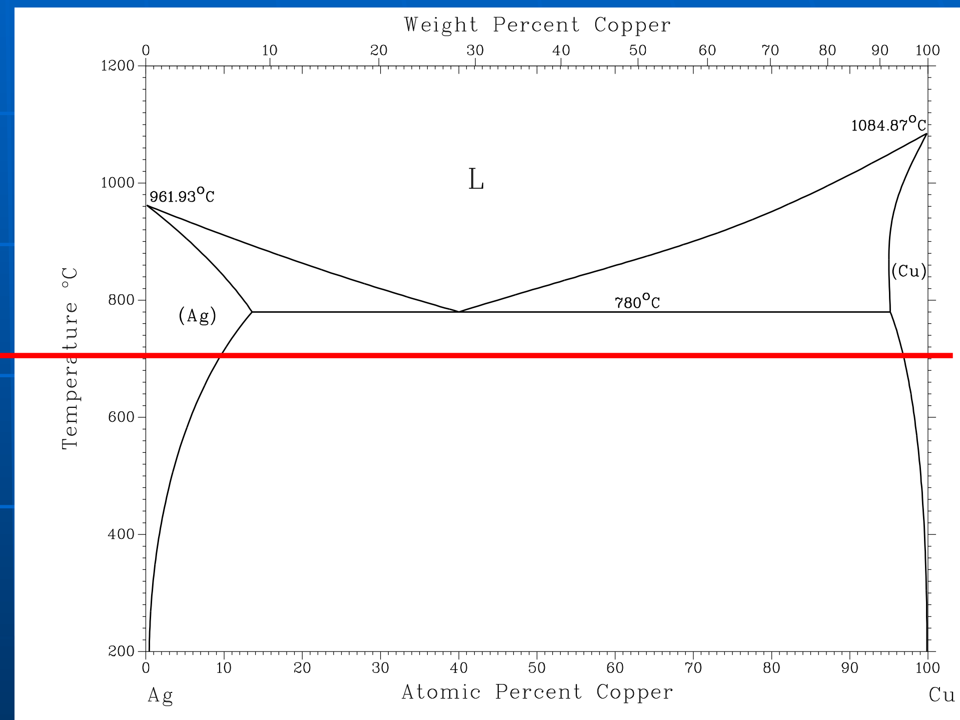


Cu/Ni

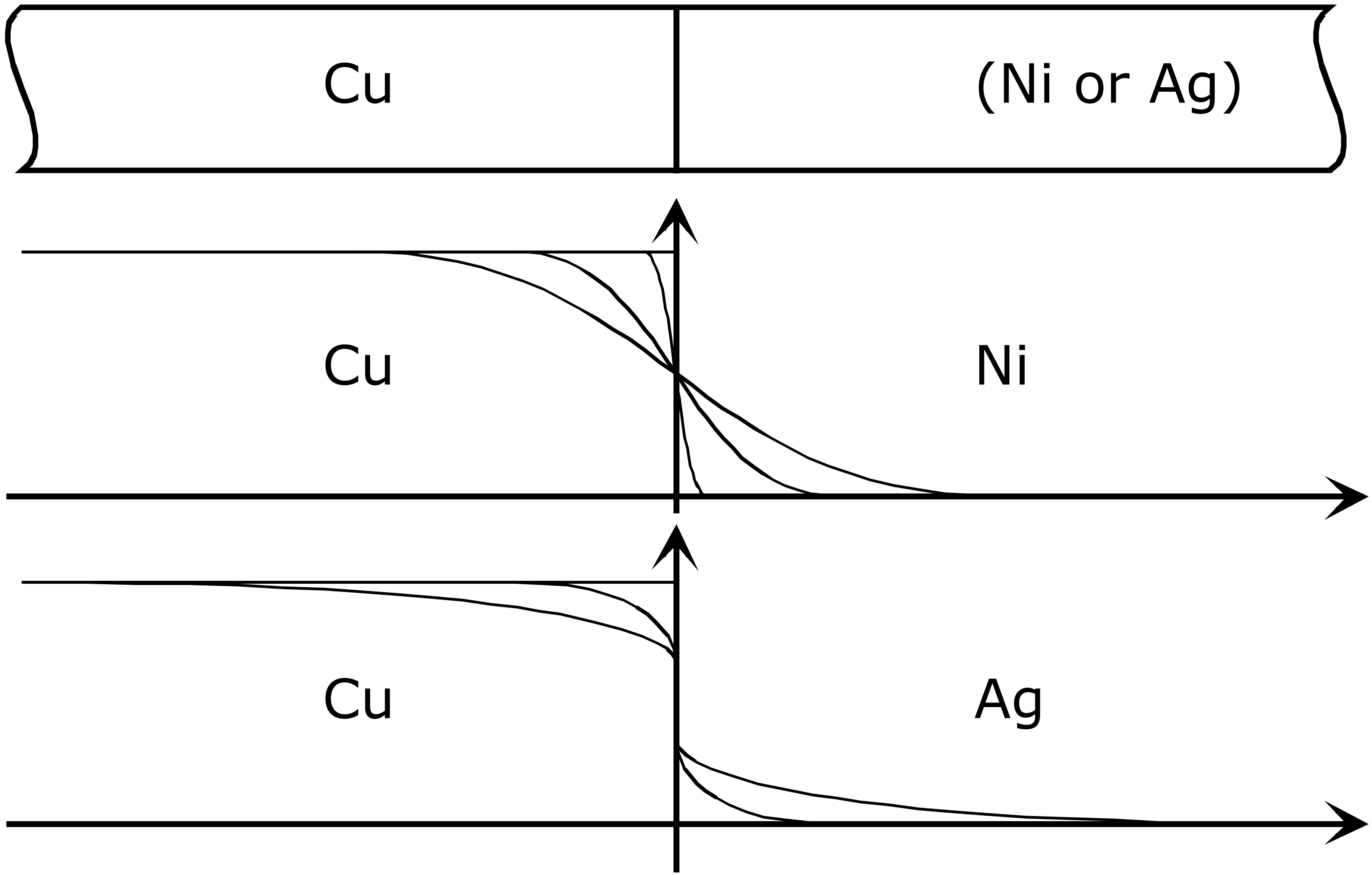


Weight Percent Nickel
L
1455°C
1084.87°C
700
(Cu,Ni)
354.5°C
67.3
T_C
354.4°C
$\alpha_1 + \alpha_2$
Temperature °C
Atomic Percent Nickel
Cu
Ni

Cu/Ag



Weight Percent Copper
L
1084.87°C
961.93°C
(Cu)
(Ag)
780°C
Temperature °C
Atomic Percent Copper
Ag
Cu


Cu
(Ni or Ag)
Cu
Ni
Cu
Ag